CHME420
Spring 2015
Final
6/19/2015 7:30 AM
Time Limit: 125 Minutes

Name (Print): ____________________

This exam contains 9 pages and 4 problems. Check for missing pages. Put your initials on the top of every page in case they become separated.

The exam is open book and notes. No cell phone use allowed during the exam.

You are required to show your work on each problem. The following rules apply:

- **Read the problem statements carefully**. Do what is requested in each problem. Show your work at every step to communicate your knowledge and thinking process.
- **Organize your work** neatly and coherently in the space provided. Scattered work without a clear order and difficult to understand will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or mathematical work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive substantial partial credit.
- If you need more space, request extra blank pages from the instructor. Label each additional blank page with your name, the problem number, and the page number for that particular problem. Place your additional pages in the right order and staple them together.

Do not write in this table:

Problem	Points	Score
1	30	
2	30	
3	20	
4	20	
Total:	100	

1. (30 points) After certain time from being lit, a cylindrical candle of radius R has length L. The wax at the top of the candle near the flame quickly reaches the melting temperature (T_m). A convective heat loss occurs from the sides of the candles to the surroundings (surrounding temperature T_a, and heat transfer coefficient h). The base of the candle is at T_a. Assume that the temperature is constant in the radial direction (the fin approximation). Your objective is to derive the temperature profile along the candle.
   (a) (4 points) Draw an appropriate shell to use in a shell balance.

   (b) (8 points) Use a shell balance to obtain the differential equation that describes the temperature variation.

   (c) (6 points) Write the boundary conditions.

   (d) (6 points) Make the differential equation and the boundary conditions non-dimensional by defining the variables $\breve{z} = z/L$ and $\Theta = \frac{T-T_a}{T_m-T_a}$.

   (e) (6 points) The solution to the differential equation is given in §C.1 of BSL. Solve for the integration constants using the boundary conditions.

   (f) (3 points (bonus)) The temperature profile in the candle reaches a steady-state quickly relative to the rate at which the candle melts (and evaporates). What is the name of an approximation we can make to obtain the time-dependence in this problem? What parameter defined above will depend on time?

   (g) (7 points (bonus)) Write the differential equation (do not attempt to solve it) that relates the height of the candle to time. Assume that there is a constant flux (q_0) entering the candle from the top in addition to the conductive flux.

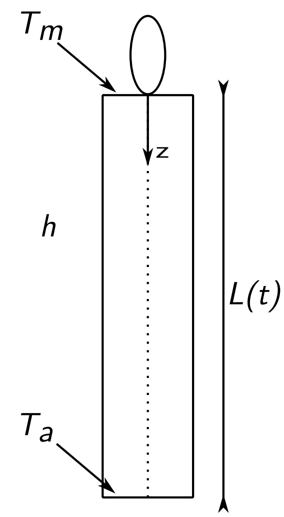

Figure 1: Candle for Problem 1.

2. (30 points) In solid tumors, O_2 is often consumed in a reaction that is nearly independent of the oxygen concentration (C_{O_2}). This means that the homogeneous reaction inside the tumor is essentially zero order (rate of reactions is k_0''' per unit volume). You may assume that the concentration of oxygen inside the tumor is small and that the concentration of O_2 at the surface of the tumor is C_R.

   (a) (8 points) Perform a shell balance to obtain the differential equation relating the concentration of oxygen to the radial coordinate in a spherical tumor.

   (b) (6 points) Write the boundary conditions for this problem.

   (c) (6 points) Show that the concentration of oxygen inside the tumor is given by (show your work carefully, now work = no points):

$$C_{O_2} = \frac{k_0''' R^2}{6D_{AB}}\left(\left(\frac{r}{R}\right)^2 - 1\right) + C_R$$

   (d) (6 points) For large enough tumors and high reaction rates, a *central* core within the tumor will stop receiving O_2, leading to cell death. What is the expression for the radius (R) of the tumor when this *first* occurs? You should use the result given in Part c) to complete this part.

   (e) (4 points) The concentration of oxygen at the surface of a brain tumor is related to the solubility of oxygen in brain tissue which is $7.14 \times 10^{-6}\,\frac{\text{moles of O}_2}{100\,\text{g of tissue}}$. The density of brain tissue is $1.05 \times 10^3\,\text{g/L}$. The ratio $D_{AB}/k_0''' = 2.22 \times 10^{-4}\,\text{m}^5/\text{mol}$. What is the radius of the tumor at which a death cell core starts to appear?

   (f) (10 points (bonus)) Find an expression for the core radius R_C as a function of the parameters in the problem.

3. (20 points) For the catalytic system described in §18.3 of the textbook an experiment is devised to measure the reaction rate around a single catalyst sphere of radius 0.5 cm. The experiment is performed at a constant temperature of 298 K and a constant pressure of 3800 mm Hg. The partial pressure of A in the bulk gas is 100 mm Hg. The measured rate of reaction is 0.0987 mol/s and the diffusivity of A in this system is 0.580 cm^2/s. Calculate the thickness of the stagnant film in the film model.

4. (20 points) A cylindrical rod of radius $R_i = 2\,\mathrm{cm}$ moves axially at a velocity of $v_0 = 0.1\,\mathrm{cm/s}$ along the central axis of a cylindrical cavity of radius $R_o = 3\,\mathrm{cm}$. The viscosity of the fluid is $\mu = 1\,\mathrm{Pa\cdot s}$ and the length of the cavity is $L = 500\,\mathrm{cm}$. The velocity profile in cylindrical coordinates inside the annular region between the cavity and the rod is:

$$\frac{v_z}{v_0} = \frac{\ln(r/R)}{\ln(\kappa)} \tag{1}$$

where κ, v_0, and R are defined in problem **2B.7** of your textbook.

(a) (15 points) Using the velocity profile given, derive an expression for the force acting on the cavity walls by the fluid.

(b) (5 points) Calculate the force acting on the cavity by the fluid. What is the direction of the force?